Max Time: 3 hours **Max Marks: 80**

N.B. (1) Question no.1 is compulsory.
(2) Use of statistical table is permitted.
(3) Figures to the right indicate full marks.

Q1. A. Solve $(D^3 - 2D^2 - 5D + 6)y = e^{3x} + 8.$ [5]

B. Using Beta and Gamma function evaluate [5]
$\int_0^2 x^2 (2 - x)^3 dx.$

C. Express into polar form and evaluate the integral [5]
$\int_0^a \int_0^{\sqrt{a^2-x^2}} e^{-(x^2+y^2)}\, dx\, dy.$

D. Evaluate $\int_1^e \int_1^{logy} \int_1^{e^x} logz\, dz\, dx\, dy.$ [5]

Q2. A. Using Beta function, Prove that $\int_0^\infty \frac{1}{1+x^2} dx = \frac{\pi}{2}.$ [6]

B. Using the method of variation of parameters, solve [6]
$\frac{d^2y}{dx^2} + a^2 y = secax\ .$

C. Show that the area between the parabolas [8]
$y^2 = 4ax \text{ and } x^2 = 4by \text{ is } \frac{16}{3}$ab.

Q3. A. Solve $(D^3 - 2D^2 - 5D + 6)y = e^{3x} + 8\ .$ [6]

B. Using Beta and Gamma function evaluate [6]
$\int_0^2 x^2 (2 - x)^3 dx\ .$

C. Change the order of integration for the integral and evaluate [8]
$\int_0^\infty \int_0^x x e^{\frac{-x^2}{y}} dxdy\ .$

Q4. A. Solve the differential equation $y\frac{dy}{dx} + \frac{4x}{3} - \frac{y^2}{3x} = 0\ .$ [6]

B. Change to polar co-ordinates and evaluate [6]
$\int_0^1 \int_0^x x + y\ dydx\ .$

C. Solve $(D^2 + 4)y = cos2x\ .$ [8]

Q5. A. Solve $(D^2 - 2D + 1)y = e^x + 1$. [6]

B. Find the length of the cardioid $r = a\,(1 - cos\theta)$ lying outside the circle $r = a\, cos\theta$. [6]

C. Evaluate $\int_0^{\frac{\pi}{6}} cos^3 3\theta\, sin^2 6\theta\, d\theta$. [8]

Q6. A. Solve $\frac{dy}{dx} + \frac{y\, cosx + siny + y}{Sinx + x\, cosy + x} = 0$. [6]

B. Find the particular integral of $(D^2 - 4D + 4)y = e^x + cos2x$. [6]

C. Find the length of the arc of the curve r = a $sin^2(\frac{\theta}{2})$ from θ = 0 to any point P(θ). [8]

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